# Applications of Mixture Models to MR Angiography

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Vector fields are a natural representation for physical phenomena. In medicine, researchers have developed techniques for generating magnetic-resonance angiography (MRA) images, in which the tissue velocity (blood flow) at all points in the volume can be represented as a three-dimensional vector field.

Phase-contrast (PC) MRA takes advantage of a flow-related effect intrinsic to magnetic resonance (MR) to image blood flow, allowing clinicians to acquire quantitative measurements of blood-flow velocity in three dimensions without using ionizing radiation or contrast agents.

We apply mixture models to reduce the noise inherent in PC MRA data and to distinguish blood vessels from background. Using the measured blood-flow velocity, and the MR-signal magnitude, our method computes the maximum-a-posteriori (MAP) estimate of the probability that a pixel within an MRA image represents flow versus background. To show potential improvements in the visualization of slow flow, we render a MAP image using maximum-intensity projection (MIP).

### Mixture modeling of flow-velocity distributions

Andersen developed a model for the distribution of flow velocities in PC MRA to improve the detectability of low-contrast vessels that exhibit slow flow [Andersen, A.H., Analysis of noise in PC MR imaging, *Med. Physics*, 23:857–869, 1996]. Let $M$ represent the magnitude of flow; and set the $z$ axis to the direction of flow. Let $v_x$, $v_y$, and $v_z$ be independent normal random variables with equal variance, $\sigma$—that is, $v_x \sim N(0, \sigma)$, $v_y \sim N(0, \sigma)$, and $v_z \sim N(M, \sigma)$. Let the flow velocity $m$ be defined as $m^2 = v_x^2 + v_y^2 + v_z^2$. The probability density function of the flow velocity, $m$, is a Rician. In the case of flow, where $M > 0$,

$$f_{\mathrm{m}}(m|M, \sigma) = \left(\frac{2}{\sigma\sqrt{2\pi}}\right)\left(\frac{m}{M}\right)\sinh\left(\frac{mM}{\sigma^2}\right)e^{\frac{-(m^2+M^2)}{2\sigma^2}}. \quad (1)$$

In the case for background, where $M=0$,

$$f_{\mathrm{m}}(m|\sigma) = \left(\frac{2}{\sigma^3\sqrt{2\pi}}\right)m^2 e^{\frac{-m^2}{2\sigma^2}}. \quad (2)$$

These equations highlight that, although individual signal components are considered normal, their flow velocity is a Rician distributed random variable whose analytical form changes depending on whether pixels are in flow or in background. The application of mixture distributions to segment and quantify MR data is not new [Santago, P. Quantification of MR brain images by mixture density and partial volume modeling, *IEEE Trans. Med Imag.*, 12:566–574, 1993]. However, these applications modeled the underlying densities of the scalar MR signal as finite mixtures of normal distributions with different means.

Our approach models the distribution of flow velocity as a finite mixture of two Ricians (Eqs. 1 and 2). The unknown parameters are the proportion of flow to background pixels ($1-\lambda$, $\lambda$), the mean flow velocity ($M_f$), the mean background noise ($M_b$), the variance of the flow velocity ($\sigma_f^2$), and the variance of the background noise ($\sigma_b^2$). To estimate the parameters for the mixture and the MAP of each pixel simultaneously, we use the class of expectation maximization (EM) algorithms [Dempster, A.P. Maximum likelihood for incomplete data via EM algorithms, *J.R. Stat. Soc.*, 39:1–38, 1977].

### Parameter estimation for the mixture of Ricians

We assume that (1) the random error in each of the components of velocity observations is normal, (2) the velocities represented are either background or flow, and (3) the velocity distributions are independent of the spatial order of the pixels within the image. The algorithm proceeds as follows.

Initialize unknown parameters to their estimates (corresponding mean and variance). At each iteration $t$ and for each pixel $i$, while

$\log(p(M_b^{(t)}, \sigma_b^{(t)} | m)) \neq \log(p(M_b^{(t+1)}, \sigma_b^{(t+1)} | m))$,

let $p(\text{pixel} | m, M_b^{(t)}, \sigma_b^{(t)}) = w_i^{(t)}$; compute

$$w_i^{(t+1)} = \frac{\lambda^{(t)}(f_{\mathrm{m}}(m|\sigma_b))}{\lambda^{(t)}(f_{\mathrm{m}}(m|\sigma_b)) + (1-\lambda^{(t)})(f_{\mathrm{m}}(m|(M_f, \sigma_f)))},$$

$\lambda^{(t+1)} = \left(\frac{1}{n}\right)\sum(1 - w_i^{(t+1)})$, where $n$ is the number of pixels, and compute the mean and variances using the maximum-likelihood estimates for the aforementioned Rician distributions.

### Preliminary results

We evaluate the accuracy of the mixture modeling for PC MRA by measuring how well the analytical model fits the acquired data. We use a synthesized dataset and two phantoms. The actual and EM-computed values for the parameters ($\lambda$, $M_f$, $\sigma_f$, $M_b$, $\sigma_b$) are

Synthesized image: (0.765,18.085,7.095,0.334,0.147),
(0.771,18.500,6.721,0.353,0.311)

Tube phantom: (0.998,136.922,67.889,7.540,4.390),
(0.999,142.116,64.829,7.542,6.172)

Carotid-bifurcation phantom:
(0.831,240.371,98.577,29.274,18.836),
(0.832,240.406,99.257,29.463, 29.925).

These results show that the distribution of MR measured flow velocity can be modeled using a finite mixture of Ricians, assuming at least two pixel types. Subjective comparison of MIP images shows that areas of slow flow that are obscured by higher-magnitude noisy pixels in threshold-segmented magnitude-weighted PC MRA images are enhanced in the threshold-segmented MAP images.

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